Modeling Iran's Tandem Cascade Configuration for Uranium Enrichment by Gas Centrifuge

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INMM 54th Annual Meeting
July 14-18, 2013 – JW Marriott Desert Springs – Palm Desert, California USA

Abstract

Presently, the gas centrifuge arranged in cascades is the most common method for enriching uranium in its fissionable isotope, ^{235}U. The enriching process can be modeled in two overarching steps. First, one examines the individual centrifuge machines, estimating enriching performance based on operating conditions such as the centrifuge feed flow rate and cut. Second, this information is built into a cascade analysis which considers the cascade structure (how the machines are linked together) as well as the total cascade feed, product, and waste flow rates. This paper, which focuses on the second half of the modeling process, outlines a method for analyzing one centrifuge cascade structure recently developed and implemented by Iran. The novel configuration consists of two conventional cascades interconnected in a tandem fashion. In this arrangement, the waste flow from the first cascade enters the second cascade as feed flow. The waste leaving this (the second) cascade becomes the overall waste flow and the associated product is reintroduced into the first cascade. Because the two cascades are linked – the flow leaving the first cascade affects the operation of the second cascade, and vice versa – the traditional method for analyzing single cascades is inadequate. This paper presents an extension of the conventional analysis procedure that is appropriate for tandem cascades. The new method is used to assess Iran's tandem cascades, including their utility for producing highly-enriched uranium for nuclear weapons.

Introduction

In the summer of 2010, Iran introduced a new centrifuge cascade design at the Pilot Fuel Enrichment Plant (PFEP) at its Natanz enrichment facility. The innovation was to join two 164-machine cascades of IR-1 centrifuges in a tandem fashion. In essence, a tandem cascade pair is a simple tails recycling strategy. The arrangement joins two cascades in such a way that the second receives as its feed the waste from the first. The second cascade's product is then reintroduced into the first cascade at whichever stage will minimize isotopic mixing.

This strategy has allowed Iran to use its existing cascades to produce near 20 percent enriched uranium hexafluoride (UF_6) from its stored 3.5 percent enriched material in such a way that the feed is stripped to an isotopic concentration near that of natural uranium (approximately 0.7 percent). Since that time, Iran has implemented two tandem sets of paired 17-stage, 174 machine cascades at its underground Fordow enrichment facility – also using them to produce near 20 percent enriched UF_6 – and, as of May 2013, has installed but not operated what may become six additional tandem pairs at the same location.

Tandem cascade analysis method

This section describes a method for estimating the performance of centrifuges arranged in tandem cascades. The hope is to achieve performance estimates that are more realistic than simple separative work calculations by accounting for the limitations imposed by real cascade structures. Even so, in general, the scarcity of open-source information precludes especially detailed or robust analyses. It is important to remember that cascade analysis techniques of this sort, even the best ones, rely on numerous assumptions.

Figure 1 gives a simple schematic for a tandem cascade. The stage numbering scheme adopted in this paper is as follows: stage one is the bottom of the secondary cascade (labeled Cascade 2), and the numbering proceeds sequentially through Cascade 2 and then through the initial cascade (labeled Cascade 1) such that the highest stage number is the top of Cascade 1. By this convention, the waste stream from the tandem cascade exits from stage one and the product stream exits from the stage with the highest number.

The stage numbers are reported in two ways. The boldface numbering system is generalized and relies on the following definitions:

Stage A – feed stage of Cascade 2;
Stage B – product stage of Cascade 2;
Stage C – first feed stage of Cascade 1, also the overall feed point; (1)
Stage D – second feed stage of Cascade 1;
Stage E – product stage of Cascade 1, also the overall product stage.

The second set of stage numbers (below the generalized ones) describes the particular tandem configuration that Iran is believed to have implemented the Natanz PFEP, where it has paired two 15-stage, 164 machine cascades. Specifically, for the tandem pair at Natanz: A is 7, B is 15, C is 21, D is 26, and E is 30. This information was obtained by the Institute for Science and International Security (ISIS).

The analysis method presented here is for two symmetric, countercurrent centrifuge cascades arranged in the tandem fashion. It relies on the concept of an ideal cascade, which is defined as a cascade for which the separation factor is constant in every stage – an idealization never achieved in practice – and for which the internal flow rates are chosen to prevent mixing between streams with unequal isotopic concentrations. The analysis method is roughly summarized with a two-step progression: first, the internal flow rates and the intermediate enrichment levels in every stage are determined for an ideal cascade of the same number of stages; then, real centrifuge performance is estimated in each stage based on the flow rates specified in the previous step. The estimated centrifuge performance, which depends on the feed rate (L) and cut (θ) of the centrifuges in the stage, is used to predict the actual enriching characteristics of the cascade.

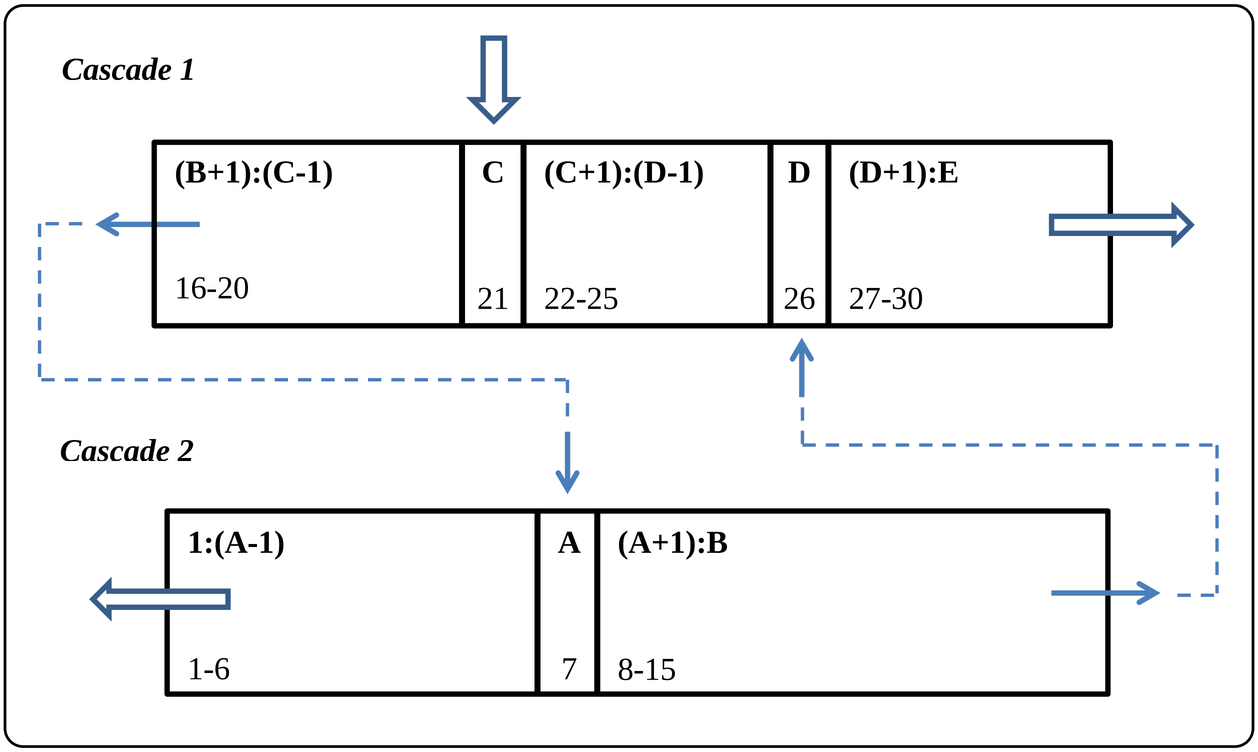

Figure 1. *When two cascades are paired in tandem, Cascade 2 recycles the tails (waste) generated by Cascade 1. The generalized stage numbering scheme is given in bold; see (1). Below it is the expected configuration of the Iranian tandem cascade at Natanz, which was obtained by the Institute for Science and International Security.*

The stage separation factor is constant for an ideal cascade. Generally, a cascade operates with a constant, predetermined feed concentration, along with a target product enrichment level. This information together with the number of enriching stages – those stages above and including the feed stage – can be used to determine the cascade's ideal stage separation factor. This factor for Cascade 1 is given by

$$\gamma_1 = \exp\left\{\frac{2\ln(R_P/R_F)}{S_{E1}}\right\}, \tag{2}$$

where R_F is the abundance ratio of the feed, R_P is the target abundance ratio of the product, and $S_{e1} = E - C + 1$ is the number of enriching stages in Cascade 1 given in terms of the parameters defined in (1).

The intermediate stage concentrations for an ideal cascade are determined easily from the feed enrichment level and the ideal stage separation factor. They may be calculated sequentially for Cascade 1 using the ideal heads and tails separation factors, α_1 and β_1, respectively, where

$$\alpha_1 = \beta_1 = \sqrt{\gamma_1}\,. \tag{3}$$

The assumption that α_1 and β_1 are equal is true in general only for an ideal cascade. After the stage concentrations are determined for Cascade 1, the process is repeated for Cascade 2. A new stage separation factor is required for Cascade 2 to prevent mixing, given by

$$\gamma_2 = \exp\left\{\frac{2\ln\left(R_D/R''_{B+1}\right)}{S_{E2}}\right\}, \tag{4}$$

where $R''_{B+1} = R_F(\beta_1)^{-(C-B)}$ is the abundance ratio of the waste stream leaving Cascade 1, $R_D = R_F(\alpha_1)^{D-C}$ is the abundance ratio of the stream entering stage D, and $S_{E2} = B - A + 1$ is the number of enriching stages in Cascade 2.

After determining the stage enrichments for an ideal cascade, the next step is to calculate the internal flow rates that must be maintained to prevent mixing. These flows depend on the overall cascade feed rate which is guessed initially but is altered later in an iterative process. The flow rate calculations follow from two simple principles: conservation of mass and conservation of species (isotope). In each stage, the following relationships must be satisfied:

$$L = L' + L'' , \tag{5}$$

and

$$NL = N'L' + N''L'' , \tag{6}$$

where L is a flow rate, N is a mass concentration of ^{235}U, and where an unprimed variable represents the feed stream, a single prime denotes the product stream, and a double prime denotes the tails stream. The solution of the resulting system of equations gives the ideal flow rates, which are described by two parameters for each stage: the feed rate (L) and cut (θ). These flow rates or slight variations thereof are used for the remainder of the analysis.

Table 1 gives the "operating line" equations for the generalized tandem cascade shown in Figure 1. This set condenses the principles outline in (5) and (6) into a single equation for each stage by eliminating the tails flow variable (L''). In addition, the equations are rearranged and simplified in recognition of the fact that many flows may be written as a combination of F, L'_E, and L'_B. Note that N_M is defined by

$$MN_M = FN_F - L''_{B+1}\, N''_{B+1} , \tag{7}$$

where $L''_{B+1} = F - M$ and $M = L'_C - L''_{C+1} = L'_E - L'_B$. The equations in Table 1, solved simultaneously, give the upflow rate (L') in every stage. The corresponding feed rates and downflow rates are determined from (5) and Figure 1.

In practice, a constant separation factor is not realized in every stage. A centrifuge's separation factor depends on the flow maintained in the machine, particularly the throughput and cut. Accordingly, the next step in the analysis process is to estimate the actual separation factors that are achieved in each stage based on the flow rates through the centrifuge machines. These predictions require a centrifuge performance map, which gives the expected separation factor as a function of feed flow and cut. A performance map may be developed from an analytical technique such as that described in [1] or by the semi-empirical method described in [2]. This paper employs the latter method to estimate the performance of the IR-1 centrifuges.

		Equation	*Applicable Stages*
Cascade 2	Stripping Section	$(F - L'_E)(N''_{s+1} - N''_1) = L'_s(N'_s - N''_{s+1})$	1:(A-1)
	Feed Point 1	$(F - L'_E)(N''_{B+1} - N''_1) = L'_B(N'_B - N''_{B+1})$	A
	Enriching Section	$L'_B(N'_B - N''_s) = L'_{s-1}(N'_{s-1} - N''_s)$	(A+1):B
Cascade 1	Stripping Section	$(F + L'_B - L'_E)(N''_{s+1} - N''_{B+1}) = L'_s(N'_s - N''_{s+1})$	(B+1):(C-1)
	Feed Point 1	$(L'_E - L'_B)(N_M - N''_{B+1}) = F(N_F - N''_{B+1})$	C
	Midsection	$L'_s(N'_s - N''_{s+1}) = (L'_E - L'_B)(N_M - N''_{s+1})$	(C+1):(D-1)
	Feed Point 2	$L'_B(N'_B - N_M) = L'_E(N'_E - N_M)$	D
	Enriching Section	$L'_E(N'_E - N''_s) = L'_{s-1}(N'_{s-1} - N''_s)$	(D+1):E

Table 1. *The operating line equations express conservation of mass and conservation of species in every stage. They are solved as a system to determine the flow rates required to avoid mixing.*

Improved predictions for the stage enrichments are made by re-solving the operating line equations and enrichment equations using the separation factors extracted from the centrifuge performance map (in place of the ideal separation factor). Typically, this correction alters the anticipated enrichment level of the product. To compensate, the cascade feed rate is varied in an iterative process until the product enrichment level is once again the target value.

Tandem cascades in Iran

Since the summer of 2010, Iran has operated one set of tandem cascades at its Pilot Fuel Enrichment Plant (PFEP) at the Natanz enrichment facility. The cascades have 15 stages each and are both composed of 164 IR-1 centrifuges. Iran introduces 3.5 percent UF_6 into the tandem set to produce near 20 percent UF_6, stripping the feed to an enrichment level near 0.7 percent.[1] The historical performance of this tandem set since mid-September of 2011 – estimated by Iran and reported by the IAEA – is summarized in Table 2. Iran has maintained fairly steady feed and product rates in these cascades over that period.

[1] The IAEA last reported the enrichment level of Iran's low-enriched uranium in October 2010. This paper assumes that value has remained near 3.5 percent.

First Day	Last Day	Duration	Feed/ Month *kg UF_6*	Product/ Month *kg UF_6*	Tails Assay *%*	Sep. Power *SWU/yr*	Sep. Power per Cent.
9/14/2011	10/27/2011	44	31.0	4.2	1.0	238	0.7
10/28/2011	2/10/2012	106	34.6	4.5	1.1	249	0.8
2/11/2012	5/17/2012	97	32.9	4.6	0.8	284	0.9
5/18/2012	8/20/2012	95	30.3	4.5	0.7	305	0.9
9/16/2012	11/10/2012	56	31.3	4.5	0.8	282	0.9
11/11/2012	2/11/2013	93	28.9	4.1	0.8	261	0.8
2/12/2013	5/9/2013	87	31.1	4.5	0.7	296	0.9
Time Weighted Average			31.6	4.4	0.8	275	0.8

Table 2. *Periodic IAEA safeguards reports give some of the enriching characteristics of the PFEP tandem cascade. It has performed fairly consistently since the fall of 2011.*

Figure 2 gives performance estimates made by a cascade model constructed with the procedure described above. The key to interpreting the figure is to recognize that the model was constrained to enrich 3.5 percent feed to 19.75 percent in every case. To maintain these enrichment goals, changes in the feed rate (the x-axis) were balanced by small modifications to the internal flow rates (not shown). Stated again, as the feed rate was altered, the flow rates (which began as the flows for an ideal cascade) were adjusted slightly in order to maintain the desired product enrichment. This process generates a range of plausible operating scenarios for the same cascade, all of which produce near 20 percent product from 3.5 percent feed.

The relationships between feed rate, product rate, separative power, and tails assay are visible in Figure 2. Not surprisingly, any increase in the feed rate drives an increase in the product flow and in the tails assay. The peak in the separative power curve corresponds to the operating scenario which is estimated to maximize the cascade's separating efficiency. The dotted lines on Figure 2 originate from a feed rate of 31.6 kg UF_6 per month, which is approximately the rate at which Iran has introduced feed into the tandem cascade at the PFEP.

Table 3 compares observed performance characteristics of the PFEP tandem cascade with predictions made with the cascade model. The modeled results were generated with a feed rate of 31.6 kg UF_6 per month and correspond to the dotted lines in Figure 2. The middle columns of the chart compare the product rate, tails assay, and separative power. The two rightmost columns compare the waste concentration of Cascade 1 and the product concentration of Cascade 2. The expected values (1.2 percent and 11 percent, respectively) were obtained by the Institute for Science and International Security.

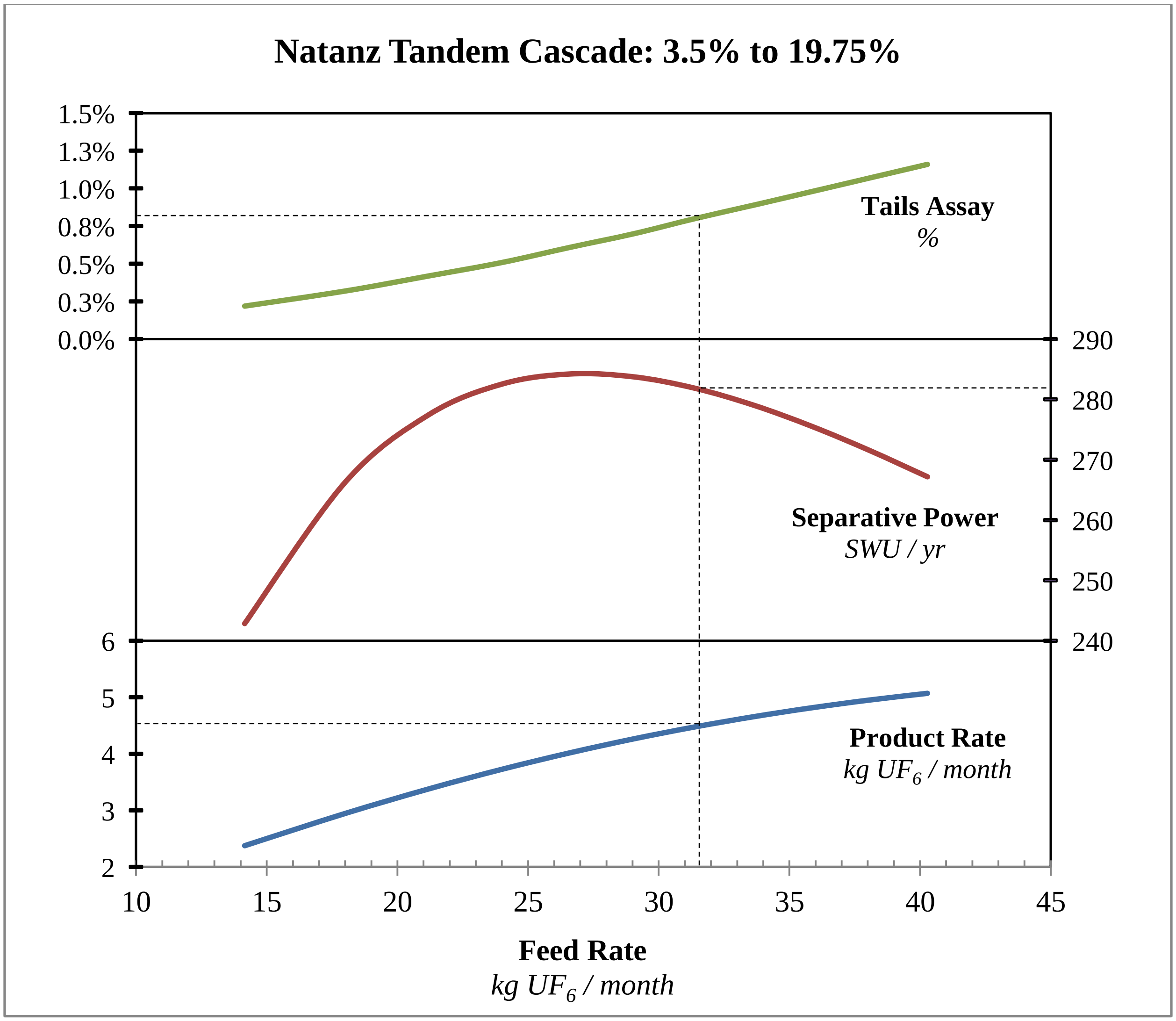

Figure 2. *Product rate, separative power, and tails assay are predicted as a function of feed rate. A range of possible operating scenarios are shown for the tandem pair, all of which enrich 3.5 percent feed to near 20 percent product.*

The strong agreement shown in Table 3 between the cascade model and the actual performance is at first surprising given the many approximations and uncertainties in the modeling process. The excellent results are explained in part by recalling that only four of the six external cascade variables (the feed, product, and waste rates; and the feed, product, and waste enrichment levels) are independent. That is, if four of the six are specified, the remaining two are constrained by the principles of conservation of mass and conservation of species. In the case at hand, three of the six external variables were known (the feed rate, feed concentration, and product concentration), leaving only one to be determined by the model. If the feed rate were unknown, the cascade could hypothetically be operating at any point on the performance curves shown in Figure 2.

	Feed *kg UF_6/ month*	Product *kg UF_6/ month*	Tails Assay *%*	Separative Power *SWU / yr*	Cascade 1 Waste *%*	Cascade 2 Product *%*
PFEP Tandem Cascade Performance	31.6	4.4	0.83	275	1.2	11
Model Performance Using Identical Feed Rate	31.6	4.5	0.81	281	1.8	10

Table 3. *The observed performance of the tandem pair at Natanz is compared to the operating scenario with the same feed rate developed by the cascade model.*

Similar cascade models were developed for three additional cascades configurations: the paired 17-stage tandem cascades at Fordow, as well as the 15-stage and 17-stage single cascades at the Fuel Enrichment Plant (FEP) at Natanz. A few additional assumptions were required to develop these models.[2,3] Using these assumptions, the predictions made by each model were compared with information given in the IAEA safeguards reports. The agreement in each case was good, and of a similar degree to that shown in Table 3.

Tandem cascades and highly-enriched uranium

One important question is whether Iran's tandem cascades would enable it to more effectively produce highly-enriched uranium for a nuclear weapon. Because tandem cascades involve a measure of tails recycling, one hypothesis is that they would be especially beneficial if Iran chose to pursue a nuclear weapon with only a small amount of near 20 percent feed material. It is often assumed that a centrifuge's enriching performance does not depend on the feed concentration, i.e. a machine that achieves a certain separation factor for low-enriched feed will achieve the same separation factor with medium- or highly-enriched feed. The authors have concluded from cascade models that, under this assumption, tandem cascades would be effective in raising near 20 percent enriched UF_6 to 60 percent enriched UF_6, and in raising 60 percent enriched UF_6 to 90 percent enriched UF_6. The results for these and other "break out" scenarios are given in [3].

Another important question is whether Iran could use its tandem cascades to produce 90 percent enriched UF_6 directly from its near 20 percent enriched stockpile, skipping the intermediate 60 percent level. Table 4 shows predictions made for both single and tandem cascades using near 20 percent enriched UF_6 as feed material. In the first two rows of the chart,

[2] In contrast to the 15-stage tandem pair at Natanz, the exact configuration of the 17-stage pair at Fordow is unknown. The Fordow tandem pairs were modeled assuming their cascades were interconnected according to the same pattern reported for the Natanz pair.
[3] Approximately 30 of Iran's roughly 54 single cascades at Natanz are 17-stage, 174-machine cascades; the rest are older 15-stage, 164 machine cascades. The IAEA only publishes the performance of the FEP facility in its entirety, which makes it impossible to determine the relative performance of the two cascade types.

the cascade models were left unaltered except for the feed enrichment; therefore, those rows give the estimated result if Iran were to leave its cascades (including all flow rates) exactly as they are, but introduce near 20 percent feed (in place of the 0.711 percent feed for the single cascades and in place of the 3.5 percent feed for the tandem cascades). The bottom rows of the chart show the approximate feed rate that would be required (and other cascade characteristics) for producing 90 percent enriched UF_6 in a single cascade or in tandem cascades.

		Feed *kg UF_6 / month*	**Product** *kg UF_6 / month*	**Product Enrichment** *%*	**Tails Assay** *%*	**Sep. Power per Cent.** *SWU / cent.-yr.*
Flows unaltered	Single Cascade	~50	~5	~65	~15	~0.7
	Tandem Cascades	~30	~5	~80	~10	~0.6
Reduced feed rate	Single Cascade	~15	~1.5	~90	~12	~0.5
	Tandem Cascades	~20	~2.5	~90	~8.5	~0.5

Table 4. *This table shows the anticipated result if Iran were to introduce near 20 percent enriched feed into its single or tandem cascades. The first two rows assume the feed rate and all internal flow rates are left unchanged. The second two rows give the estimated feed rate (note that the internal flow rates must be altered slightly) that would produce 90 percent enriched UF_6 directly from near 20 percent feed.*

In all cases, the cascade models suggest that overall separative efficiency would drop with the introduction of near 20 percent feed. This result is unsurprising; when a cascade is not being operated as designed, one expects lower separative efficiencies. Two additional observations arise from the estimates in Table 4:

1. While both single cascades and tandem cascades are capable of producing 90 percent product from near 20 percent feed, neither cascade type could perform this task without alterations. These alterations would include lowering the feed rate and adjusting valves to set proper internal cascade flows, but would not include re-piping. Iran’s tandem cascades would require a lesser degree of alteration than its single cascades, in part because they are currently enriching to a higher level than the single cascades.
2. Tandem cascades, because of their recycling qualities, would enable Iran to produce a greater amount of 90 percent enriched product with a smaller amount of near 20 percent enriched feed.

Conclusions

The primary intent of this paper was to present a generalized method for modeling tandem cascades. The method was tested by comparing its predictions to the performance of Iran’s tandem pair at the Natanz PFEP. A fair amount is known about this cascade pair, and the modeling process was able to generate predictions that agreed well with its observed performance. In situations for which less is known about a cascade, the modeling process is less able to make detailed predictions and instead produces a range of feasible operating scenarios.

Iran's stated intention for its tandem cascades is to produce near 20 percent enriched UF_6 in such a way that the waste is "reduced from ~2% to ~0.7% U-235" [4]. Cascade models suggest that the tandem configurations implemented at Natanz and Fordow are well-suited for achieving this goal. Using tandem cascades, Iran is able to meet its enrichment targets while maintaining its highest separating performance (approximately 0.9 SWU/cent.-yr.) in its IR-1 centrifuges.

Iran's tandem cascades appear to be well-designed for producing near 20 percent enriched UF_6; however, if Iran were to pursue weapons-grade uranium, its tandem cascades could be important assets. In one scenario, they would help Iran conserve medium-enriched (20 and 60 percent) material en route to weapons-grade material; see [3]. Alternatively, Iran could use its tandem cascades to produce weapons-grade material directly from its near 20 percent stockpile at an estimated rate of 2.5 kg UF_6 per month, per cascade. At this rate, one tandem cascade could, if operated continuously, produce one significant quantity (25 kg U, or about 37 kg UF_6) of weapons-grade uranium in about 15 months. Eight such tandem cascades, or the number that could potentially become operational in the Fordow enrichment facility, could do so in about 2 months. Yet, to produce one significant quantity in this manner, Iran would require roughly 300 kg of stockpiled UF_6 enriched to near 20 percent. As of May 2013, its stockpile of available near 20 percent UF_6 was only 182 kg.[4]

References

[1] H. G. Wood and J. B. Morton, "Onsager's pancake approximation for the fluid dynamics of a gas centrifuge," *J. Fluid Mech.*, vol. 101, no. 01, pp. 1-31, 1980.

[2] P. J. Migliorini, W. C. Witt and H. G. Wood, "Semi-empirical method for developing a centrifuge performance map," *Submitted to Separation Science and Technology.*

[3] W. C. Witt, C. Walrond, D. Albright and H. Wood, "Iran's Evolving Breakout Potential," Institute for Science and International Security, October 8, 2012.

[4] IAEA Board of Governors, "Implementation of the NPT Safeguards Agreement and relevant provisions of Security Council resolutions in the Islamic Republic of Iran," 23 November 2012.

[4] This figure does not include 141 kg of near 20 percent UF_6 that has been fed into the conversion process at the Fuel Plate Fabrication Plant near Esfahan.